Thus, in calculations of the heating of solid bodies in a plasma jet one must allow for the nonsteadiness of heat exchange in the stage of a transient process, the duration of which depends not only on the parameters of the body but also on the properties of the plasma jet.

NOTATION

R, radius of thermocouple junction, m; a, thermal diffusivity, m^2/sec; δ_{th}, thickness of thermal boundary layer, m; λ, thermal conductivity of gas, J/m•sec•deg; α, heat-transfer coefficient, W/m^2•deg.

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AN ANALYTICAL INVESTIGATION OF THE LONGITUDINAL TEMPERATURE PROFILE DEVELOPING DURING THE COOLING OF A CRYOGENIC PIPELINE

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Analytical expressions are obtained for the longitudinal temperature profiles of the wall and the stream of cryogen during the cooling of a cryogenic pipeline. A comparison of the calculated data with experiment gives their good agreement.

To determine the nonsteady temperature fields developing during the cooling of a pipeline one used [1] a one-dimensional description of heat transfer, it being assumed that the flow velocity of the cryogen in a given cross section is constant while the temperature only varies along the length of the pipeline.

If one neglects heat conduction of the cryogen toward the wall of the pipeline in the longitudinal direction and considers the case when the ratio of the heat capacities of the cryogen and the wall of the pipeline per unit length is small, the cooling will be described by the system of equations [2]

$$\begin{cases} (cG)_g \dfrac{\partial T_g}{\partial x} = \alpha \Pi (T_w - T_g); \\ (Fc\rho)_w \dfrac{\partial T_w}{\partial t} = \alpha \Pi (T_g - T_w) \end{cases}$$

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or, in dimensionless form,

$$\begin{cases} \dfrac{\partial T_w}{\partial \eta} = A(T_g - T_w); \\ \dfrac{\partial T_g}{\partial \xi} = B(T_w - T_g), \end{cases} \tag{1}$$

where

$$\eta = \frac{\alpha|_{T_{g0}} \Pi}{(\rho c F)_w} t; \qquad \xi = \frac{\alpha|_{T_{g0}} \Pi}{(c_g G)_0} x, \tag{2}$$

$$A = \frac{\alpha}{\alpha|_{T_{g0}}} \frac{c_{w0}}{c_w}; \quad B = \frac{\alpha}{\alpha|_{T_{g0}}} \frac{G_0}{G(\eta)}; \tag{3}$$

$$\frac{\alpha}{\alpha|_{T_{g0}}} = \frac{\lambda_g}{\lambda_{g0}}. \tag{4}$$

It should be noted that Eq. (4) is valid in the case of steady heat exchange in the laminar mode of flow. Since the cooling of a cryogenic pipeline is a nonsteady process, nonsteadiness parameters [1] must be introduced into (4). As shown by estimates made for the cooling modes considered in the present work, however, allowance for these parameters does not lead to a pronounced change in α.

A finite-difference algorithm for the numerical solution of the system (1), developed and realized on a computer, is described in [2], and a calculation is also made of the nonsteady temperature fields and cooling time of a coaxial cable with allowance for the variability of the operating and physical parameters.

From an analysis of [2-4] it follows that in a sufficiently long pipeline being cooled one can distinguish a completely cooled and a completely uncooled section, between which there is a zone of heat exchange between the cryogen and the wall of the pipeline where the temperature varies from T_{g0} to T_{w0}. Strictly speaking, the wall temperature never reaches the value T_{g0} but will only approach infinitely close to it. But one can assign a temperature interval from $(T_{w0} - \varepsilon)$ to $(T_{g0} + \varepsilon)$, where ε is any temperature change as small as desired, outside of which the cooling process is approximately considered as fully completed. This temperature interval bounds the zone of heat exchange, which will have a fully defined width for a concrete value of ε. In the initial stages of cooling the zone of heat exchange forms and starts to move along the pipeline with a certain velocity. The cooling of the pipeline is completed at that moment when the zone of heat exchange passes entirely through the pipeline.

In [5] the hypothesis is advanced that after a certain time following the start of cooling with a constant flow rate of cryogen a temperature profile forms in the zone of heat exchange which does not change its shape with time and moves with a constant velocity v_1:

$$v_1 = d\xi/d\eta = \text{const.} \tag{5}$$

When the condition (5) is satisfied the system of equations (1) can be integrated, and one can obtain an analytical expression for the steady temperature profile.

With allowance for (5) we introduce a new coordinate system moving with the velocity v_1, the origin of which is placed at the point

$$T_w = \frac{1}{2}(T_{w0} - T_{g0})$$

on the temperature profile of the wall.

The connection between the old and new coordinate systems is expressed by the equation

$$\chi = \xi - v_1\eta + C, \tag{6}$$

where C is some constant which does not appear in the subsequent calculations.

In the new coordinate system the temperature is a function not of the two variables ξ and η, as in the old system, but of the one variable χ, which in turn depends on ξ and η in accordance with Eq. (6). In accordance with this we obtain

$$\frac{\partial T}{\partial \xi}=\frac{\partial T}{\partial \chi}\frac{\partial \chi}{\partial \xi}=\frac{dT}{d\chi};\quad \frac{\partial T}{\partial \eta}=\frac{\partial T}{\partial \chi}\frac{\partial \chi}{\partial \eta}=-v_1\frac{dT}{d\chi}.$$

In this case the system of equations (1) is converted to the form

$$\begin{cases} v_1\dfrac{d\theta_w}{d\chi}=A(\theta_w-\theta_g); \\ \dfrac{d\theta_g}{d\chi}=B(\theta_w-\theta_g), \end{cases} \tag{7}$$

where

$$\theta_w=\frac{T_w-T_{g0}}{T_{w0}-T_{g0}};\qquad \theta_g=\frac{T_g-T_{g0}}{T_{w0}-T_{g0}}.$$

The system of equations (7) determines the temperature profiles of the wall $\theta_w = f_1(\chi)$ and the cryogen $\theta_g = f_2(\chi)$ in the steady zone of heat exchange, which can be calculated knowing the dependences of the heat capacity of the wall and the heat-transfer coefficient on the temperature and of the flow rate of cryogen on time. With a quadratic dependence of the heat capacity of the wall on the temperature, a linear dependence of the heat-transfer coefficient on θ_w, and a constant flow rate of cryogen it is easy to obtain an analytical expression determining the temperature profile. And this will be done below.

From the system of equations (7) with allowance for (3) at a constant flow rate of cryogen, it follows that

$$\frac{d\theta_g}{d\chi}=\frac{c_w}{c_{w0}}v_1\frac{d\theta_w}{d\chi}. \tag{8}$$

From this, assuming that $\theta_g = \theta_w$ at the start and end of the steady zone of heat exchange, we can find the velocity

$$v_1=c_{w0}\Big/\int_0^1 c_w d\theta_w=c_{w0}/\bar{c}_w,$$

where $\bar{c}_w$ is the integral-mean value of the heat capacity of the wall in the temperature interval of θ_w from zero to one.

The quadratic dependence of the heat capacity of the wall and the linear dependence of the heat-transfer coefficient on the temperature can be written in the following general form:

$$c_{wr}\equiv c_w/c_w|_{T_{g0}}=1+a\theta_w+b\theta_w^2;\quad \alpha_r\equiv\alpha/\alpha|_{T_{g0}}=1+m\theta_w. \tag{9}$$

After substituting Eqs. (3), (8), and (9) into the system of equations (7) with a constant flow rate we obtain an integral expression connecting the dimensionless coordinate χ with the dimensionless wall temperature θ_w:

$$\chi=-\int_{0,5}^{\theta_w}\frac{(1+a\theta_w+b\theta_w^2)\,d\theta_w}{\theta_w(1+m\theta_w)\left[\dfrac{b}{3}\theta_w^2+\dfrac{a}{2}\theta_w+(1-\bar{c}_w/c_w|_{T_{g0}})\right]}.$$

After integration we obtain the following dependence between χ and θ_w:

$$\chi=D\ln\frac{0,5}{1-\theta_w}-\frac{2}{a(k+1)}\ln\frac{0,5}{\theta_w}+\frac{1}{m(k+1)-k}\left\{k\left[3+\frac{2}{a(k+1)}-D(m+1)\right]\ln\left|\frac{\theta_w k+(k+1)}{1,5k+1}\right|+\right.$$
$$\left.+\left[3k+\frac{2m}{a}-m(2k+1)D\right]\ln\left|\frac{0.5m+1}{\theta_w m+1}\right|\right\}, \tag{10}$$

where

$$D=\frac{\left(2+3k+\dfrac{2}{a}\right)[m(k+1)-k]}{(2k+1)[m+m^2(k+1)-k]};\quad k=\frac{2b}{3a}. \tag{11}$$

By integrating Eq. (8) with allowance for (9) we can obtain the connection between the dimensionless stream temperature and the dimensionless wall temperature:

$$\theta_g = \frac{\theta_w + \frac{a}{2}\theta_w^2 + \frac{b}{3}\theta_w^3}{1 + \frac{a}{2} + \frac{b}{3}}. \tag{12}$$

In the derivation given above the flow rate of cryogen was taken as constant. However, the flow rate varies along the length of the channel even in the case when it is kept constant at the channel entrance. We can show that this variation is small in the cooling of a copper cryogenic pipeline by helium.

We will take the flow rate of cryogen at the entrance to the pipeline as constant, the cryogen as an ideal gas, and the pressure as constant along the length of the channel. Then from the Mendeleev–Clapeyron equation it follows that

$$\rho_g = \frac{P}{RT_g} = \frac{\text{const}}{T_g},$$

where P is the cryogen pressure in the channel; R is the gas constant for the cryogen.

It is natural that in the zone of heat exchange, along with the steady temperature profile there also forms a steady density profile, which moves with the same constant velocity v_1.

Using (2) we write the continuity equation:

$$\frac{G_0 c_{g0} F_g}{(\rho c_0 F)_w} \frac{\partial \rho_g}{\partial \eta} + \frac{\partial G}{\partial \xi} = 0. \tag{13}$$

In considering the motion of the density profile, we find the connection between the derivatives $\partial\rho_g/\partial\eta$ and $\partial\rho_g/\partial\xi$:

$$\partial\rho_g/\partial\xi = -(1/v_1)(\partial\rho_g/\partial\eta). \tag{14}$$

We transform Eq. (13), and with allowance for (14) we have

$$-\frac{G_0 c_{g0} F_g}{(\rho c_0 F)_w} v_1 \frac{\partial \rho_g}{\partial \xi} + \frac{\partial G}{\partial \xi} = 0.$$

Integrating this expression over the coordinate ξ, we will have

$$G - G_0 = \frac{G_0 c_{g0} F_g}{(\rho c_0 F)_w} v_1 (\rho_g - \rho_{g0}),$$

from which

$$G_r \equiv \frac{G}{G_0} = 1 + \frac{c_{g0} F_g v_1 \rho_{g0}}{(\rho c_0 F)_w}\left(\frac{\rho_g}{\rho_{g0}} - 1\right). \tag{15}$$

Equation (15) shows that in the zone of heat exchange G_r decreases from one to

$$(G_r)_{\min} = 1 + \frac{(c_0\rho_0 F)_g}{(\bar{c}\rho F)_w}\left(\frac{T_{g0}}{T_{w0}} - 1\right).$$

For the experimental installation considered in the present report with cooling from 300 to 20°K and a pressure of 0.4 MN/m² we have $(G_r)_{\min} = 0.966$. The values of c_w are taken from [6] and those of c_g and ρ_g from [7].

Thus, in the example under consideration the helium flow rate in the zone of heat exchange decreases by 3.5%. We note that before and after the zone of heat exchange the flow rates of cryogen are constant but they differ by 3.5%.

The temperature profiles in the steady zone of heat exchange were calculated using Eqs. (10)-(12) for a copper pipeline cooled by helium. The temperature dependences of the heat capacity of copper and of the heat-transfer coefficient are presented in Figs. 1 and 2. The dependence $c_w(T)$ is constructed from the data of [6] and $\alpha(T)$ from (4). The thermophysical properties of helium are taken from [7] for a pressure of 4 abs. atm.

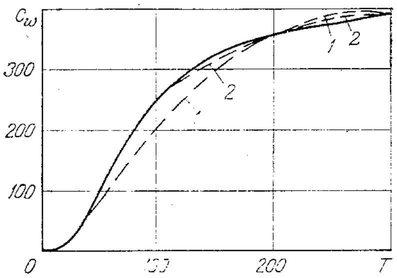

Fig. 1

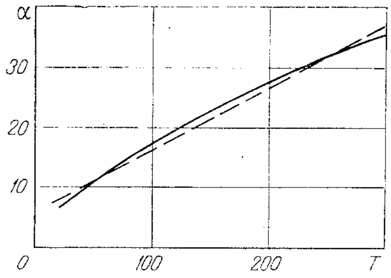

Fig. 2

Fig. 1. Variation of the heat capacity of copper c_w (J/kg•°K) with temperature T (°K) (solid line) and quadratic approximations of this dependence (dashed lines): 1) for the temperature range of 20-300°K; 2) 100-300°K.

Fig. 2. Variation of the heat-transfer coefficient α (W/m²•°K) with temperatute T (°K) (solid line) and the linear approximation of this dependence (dashed line).

The dashed lines in Fig. 1 show the quadratic approximations of the dependence $c_w(T)$ for the temperature range of 20-300°K with the coefficients a = 105 and b = −57 and for the temperature range of 100-300°K with the coefficients a = 1.11 and b = −0.58, while the dashed line in Fig. 2 shows the linear approximation of the dependence $\alpha(T)$. For the temperature range of 20-300°K the coefficient m equals 3.5, while for 100-300°K, m = 1.3.

These approximations were used in the calculations. Their maximum departure from the true c_w and α is 10%.

The temperature profiles for different $\alpha_r(\theta_w)$ are shown in Fig. 3. It is seen from the figure that the zone of heat exchange is narrower if the heat-transfer coefficient is not constant but decreases with a decrease in temperature.

Temperature profiles for different $c_{wr}(\theta_w)$ are shown in Fig. 4. For cooling in the range of 100-300°K, where the heat capacity of the wall varies less with temperature, the zone of heat exchange is found to be wider.

For a constant value of the heat capacity of the wall the zone of heat exchange does not assume a steady value regardless of the form of the temperature dependence $\alpha_r(\theta_w)$.

We can show this by using Eqs. (10) and (11).

For a linear temperature dependence of the heat capacity of the wall the coefficient b equals zero, and hence k = 2b/3a = 0. Then

$$\chi = \frac{1}{a}\left[\frac{2(a+1)}{1+m}\ln\frac{0.5}{1-\theta_w} - 2\ln\frac{0.5}{\theta_w} + \frac{2(m-a)}{1+m}\ln\left|\frac{0.5m+1}{\theta_w m+1}\right|\right].$$

The heat capacity of the wall will be constant when a = b = 0, but then the value of χ becomes equal to ∞ for any m and θ_w, and hence a steady zone of heat exchange is not formed.

The experimental verification of Eqs. (10)-(12) was performed on the 1SPK-M installation, which consists of a section of coaxial superconducting cable where the cryogen (helium) flows through the gap between the two cores, which consist of copper tubes with the superconductor deposited on their surfaces. The wall temperature was determined from the electrical resistance of individual sections of cable, measured with potential probes mounted on the outer surface of the inner tube [8]. The length of the sections was ~1 m. The total length of the pipeline was 5.6 m. Tube diameters: inner 55 × 2.5 mm, outer 80 × 3.0 mm.

The cooling was done from room temperature (300°K) to 100°K at a helium flow rate of $8 \cdot 10^{-4}$ kg/sec and to 20°K at a flow rate of $1.75 \cdot 10^{-3}$ kg/sec. The helium flow rate at the entrance to the cable was kept constant. The gas pressure was 4 abs. atm. and varied insignificantly along the length of the pipeline because of the low hydraulic resistance.

It is convenient to compare the calculated and experimental results on the width $\Delta\eta$ of the zone of heat exchange [5]. As the width of the zone of heat exchange we understand the time in which the wall temperature at some definite cross section of the pipeline varies from $(1 - \varepsilon)$ to ε.

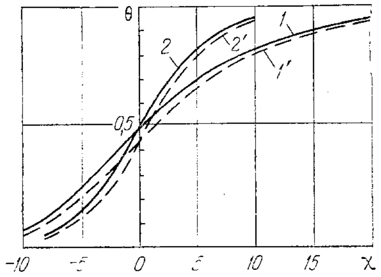

Fig. 3

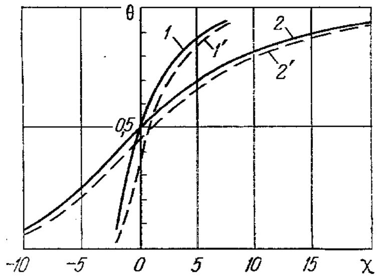

Fig. 4

Fig. 3. Temperature profiles of wall (solid lines) and cryogen (dashed lines) in the steady zone of heat exchange for different values of the heat-transfer coefficient: 1 and 1') for m = 0; 2 and 2') for m = 1.3. The heat capacity of the wall is given in the approximation 2 in Fig. 1.

Fig. 4. Temperature profiles of wall (solid lines) and cryogen (dashed lines) in the steady zone of heat exchange for different laws of variation of the heat capacity of the wall for m = 0: 1 and 1') for a law of variation of c_w corresponding to the approximation 1 in Fig. 1; 2 and 2') for a law of variation of c_w corresponding to the approximation 2 in Fig. 1.

As a result of the experiment we obtained dependences $T_w(t)$, which were reduced to the dimensionless form $\theta_w(\eta)$ using Eqs. (2) and (7). For this the quantity $\alpha|_{T_{g0}}$ was determined from the equation for the laminar mode of flow

$$\alpha|_{T_{g0}} = 4.36\frac{\lambda_{g0}}{d}, \tag{16}$$

while the value of ε was taken as $\varepsilon = (T_{w0} - T_{g0})/10$.

For cooling to 100°K the value of $\Delta\eta$ is 10.4, while $\Delta\eta = 1.9$ for cooling to 20°K. The values of the width of the zone of heat exchange calculated from Eq. (10) are $\Delta\eta = 12.0$ and $\Delta\eta = 1.7$, respectively.

Thus, the calculated and experimental results agree well with each other, which indicates that the analytical expressions obtained are suitable for calculating the cooling process.

NOTATION

T, temperature, °K; ρ, density, kg/m³; α, heat-transfer coefficient between wall and stream, W/m²·°K; Π, perimeter wetted by stream, m; c, heat capacity, J/kg·°K; F, cross-sectional area, m²; G, flow rate of cryogen, kg/sec; t, time, sec; x, longitudinal coordinate, m; λ, coefficient of thermal conductivity of cryogen, W/m·°K; μ, coefficient of dynamic viscosity, m²/sec; Pr, Prandtl number; η, dimensionless time; ξ, dimensionless longitudinal coordinate; χ, dimensionless longitudinal coordinate in the moving coordinate system; Δη, width of zone of heat exchange; θ, dimensionless temperature; P, pressure of cryogen, N/m²; R, gas constant, J/kg·°K; ε, dimensionality of temperature; v_1, dimensionless velocity of movement of steady temperature profile; $\bar{c}_w$, integral-mean heat capacity of wall, J/kg·°K; *a*, b, m, constant coefficients in the approximating equations. Indices: 0, initial value; w, wall; g, cryogen; r, relative value.

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CALCULATION OF INITIAL STAGE OF HEATING A PLANAR BODY WITH VARIABLE PROPERTIES

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UDC 536.244

A method is presented for calculation of upper and lower limits of the temperature field of a planar body with temperature-dependent thermophysical properties.

In the initial stage of heating, a planar body can be considered as semiinfinite. The differential transfer equation with consideration of temperature dependence of the thermal conductivity and specific heat can then be written in the form

$$\frac{d}{d\eta}\left[f_1(\Theta)\frac{d\Theta}{d\eta}\right]+2\eta f_2(\Theta)\frac{d\Theta}{d\eta}=0, \tag{1}$$

where $\eta = \sqrt{x^2/a_0\tau}/2$ is the Boltzmann variable, and $f_1(\Theta)$ and $f_2(\Theta)$ are positive functions which do not go to zero over the range of Θ from 0 to 1. We supplement Eq. (1) by the boundary conditions

$$\Theta = 0 \quad \text{for} \quad \eta = 0, \tag{2}$$

$$\Theta = 1 \quad \text{for} \quad \eta \to \infty. \tag{3}$$

In the general case Eq. (1) is nonlinear, so achievement of an analytical solution is difficult.

To study the problem presented by Eqs. (1)-(3), we will use the approach proposed in [1, 2], which considered nonstationary thermal conductivity of bodies with nonlinear boundary conditions. Following [1, 2], we will find upper and lower limits for the unknown temperature field Θ. This method is applicable in engineering practice when the "gap" between the limiting functions is relatively small and the equations involved are relatively simple.

We will now demonstrate the application of this principle to Eqs. (1)-(3).

Introducing the Kirchhoff substitution

$$U=\int_0^{\Theta} f_1(\Theta)\,d\Theta \Big/ \int_0^1 f_1(\Theta)\,d\Theta, \tag{4}$$

we transform Eqs. (1)-(3) to the form

$$\frac{d^2U}{d\eta^2}+2\eta\frac{f_2(\Theta)}{f_1(\Theta)}\frac{dU}{d\eta}=0, \tag{5}$$

$$U = 0 \quad \text{for} \quad \eta = 0, \tag{6}$$

$$U = 1 \quad \text{for} \quad \eta \to \infty. \tag{7}$$

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